# Surf Physics

**Ronald Edge**

Surfboard riding has an aura of "endless summer" about it — bronzed bodies with arms outstretched, balancing on a small board and riding the front of a large wave as it advances toward the shore. It was characterized as a "royal sport for the natural kings of earth" by Jack London, who described a Waikiki Beach surfer "rising like a sea-god from out of the welter of spume and churning white."[1]

The first Westerner to comment on such a captivating scene was Captain James Cook, who in 1777 described a canoe surfer in Tahiti:

> *He went out from shore till he was near the place where the swell begins to take its rise; and, watching its first motion very attentively, paddled before it with great quickness, till he found that it overlooked him, and had acquired sufficient force to carry his canoe before it without passing underneath it. He sat motionless and was carried along at the same swift rate as the wave, till it landed him upon the beach.*[2]

A year later Cook arrived on the big island of Hawaii and there witnessed the first board surfers and recorded it in his journal. The expedition's artist sketched a man on a blunt-nosed surfboard paddling out to meet the British ships.[2]

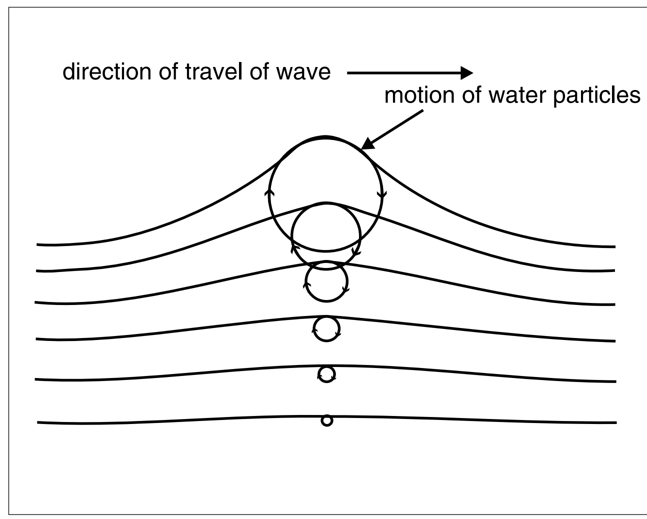


**Fig. 1. Circular motion of water particles in deep-water wave.**

Just what is happening when a surfer taps into the energy of a breaking wave and rides to shore? It's sport, it's art, it's skill, stamina, and drama. It is also physics — hydrodynamics, wave propagation, kinematics, and dynamics.

## Gravity Driven Waves on Water

The hydrodynamics of water waves is highly nonlinear, making them difficult to treat analytically. However, some of the basic concepts and some order-

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of-magnitude calculations are accessible to introductory level physics students.

***Waves in Deep and Shallow Water.*** In deep ocean water with wind-driven waves, the water "particles" rotate in a circle, forward at the crest and backward at the trough, so there is (approximately) no net motion of the water as the wave passes (see Fig. 1). For these waves, the circular motion diminishes exponentially with depth, becoming negligible at depths greater than the wavelength of the wave.[3]

The motion of the waves may be characterized by the phase velocity (the speed of one crest in a whole bunch all of the same height) and the group velocity, which is the speed of the bunch or wave train as a whole. The full equation for the phase velocity of continuous waves in water of any depth $h$ is given by[4]:

$$v^2 = (g/k)\tanh(kh),$$

where $k = 2\pi/\lambda$, $g$ is the acceleration due to gravity, and $\lambda$ is the wavelength. This reduces to $v = \sqrt{gh}$ for very shallow waves and to $v = \sqrt{g/k}$ for deep-water waves. [Deep-water waves are generally taken to be those for which the ratio of water depth to wavelength ($h/\lambda$) is greater than 0.5. For shallow-water waves, $h/\lambda < 0.005$.][5] These results indicate that the wave speed in deep water depends on the wavelength (longer waves travel faster) while in shallow water the speed is wavelength independent, varying only with the water depth. Of course, this changeover leads to very interesting effects at the beach, where the transition from deep to shallow occurs over a short distance.

The group velocity of water waves is given by[6]

$$u = \tfrac{1}{2}v[1 - 2kh\,\mathrm{cosech}(2kh)].$$

For deep-water waves, this reduces to $u = v/2$; i.e., the group velocity is half the phase velocity.

The effect of the difference between group and phase velocity for deep-water waves may be observed at the beach. Some distance from shore, one may see a large wave approaching (myth has it every wave in seven is such). This wave (think of it as a pulse) is a superposition of ordinary deep-water waves that are traveling at the (wavelength-dependent) phase velocity. It is possible to observe waves entering the pulse from the rear and advancing through it. The pulse itself moves with the slower group speed. Therefore, it takes twice as long for the big wave to arrive as might be thought by observing the speed of the individual crests, the difference of the two speeds being a factor of two.

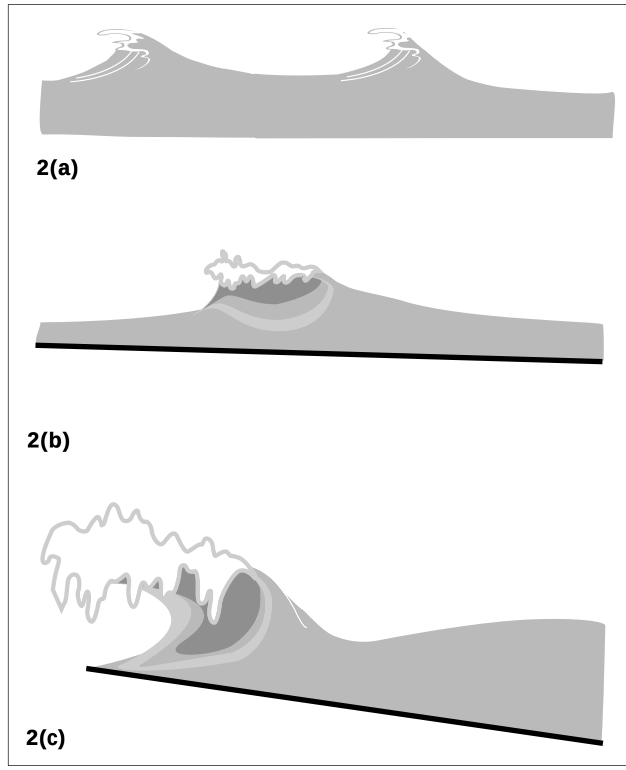


**Fig. 2. Ocean waves: (a) deep-water waves, (b) spilling breaker, and (c) plunging breaker.**

***Breaking waves.*** The physics of gigantic breaking waves, such as one sees in Hawaii or in the movies, is even more complex. The speed of the incoming wave is initially determined by its wavelength in deep water, and this in turn is dependent on the "fetch" or distance over which the wind has blown in raising the wave. Fetch distances of 900 nautical miles are thought to be sufficient for producing the largest amplitude storm waves (height $\geq$ 100 ft).[7] Clearly, the maximum wave speed cannot exceed the wind speed, but in a hurricane, this may easily be in excess of 120 mph. An empirical relationship has been developed between the fetch, the windspeed, and the mean height and wavelength of the risen sea.[8,9] Waves have been recorded with a period of 29 s, corresponding to a speed of 100 mph and a wavelength of a mile or more.[10,11] Such big waves, once risen, take a very long time to die out, and

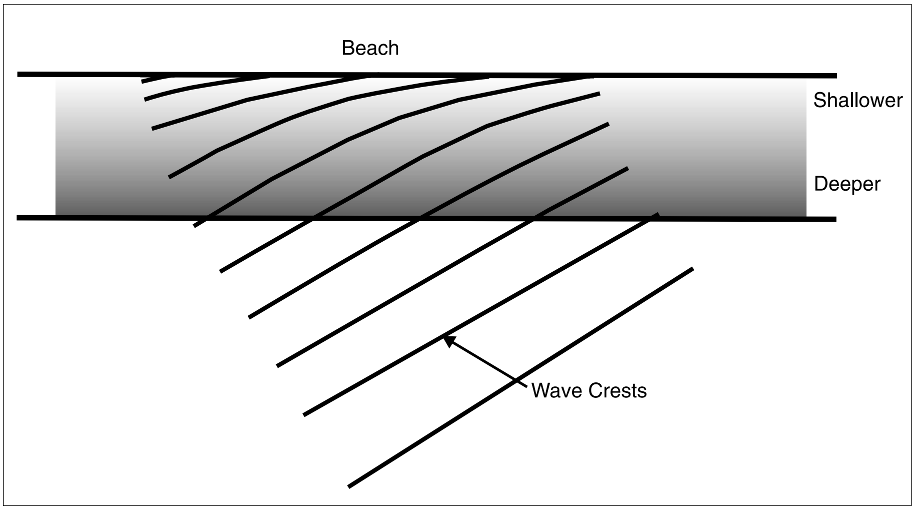


**Fig. 3. Waves refract as they approach a smooth beach having gentle slope.**

## Body Surfing

Let's begin where many young surfers begin, standing about hip-deep in water and going chest down on a "boogie board" (a Styrofoam float), an inflatable raft, or even just using one's body. You are then carried in on the front of the wave, which continues on after the wave has broken. Generally, a small wave, say about 30 cm (1 ft) in height is used. If we insert $h$ = 30 cm into the shallow water equation and $g = 9.81$ m/s$^2$, $v$ is then 1.71 m/s or 3.8 mph. Since both the rider and the boogie board can float (hopefully), they are not carried under by the wave.

these are referred to as the "swell" — big waves not in the vicinity of storms. Theory shows that in the deep sea, the wave cannot have a sharper crest angle than 120°, which is too small a slope for conventional surfing[12] [see Fig. 2(a)]. As a wave approaches the shore, the depth at the crest is larger than at the trough, where shallow-water wave theory shows the speed is lower, so the crest overtakes the trough, the peak sharpens, and the wave ultimately breaks. Figure 2 shows some varieties of breaking waves. The spilling breaker (b) is concave on both the front and back sides, and is more likely to occur if the wind is blowing out from the shore. Such a wave gradually loses its form as it advances and water spills off its crest. The plunging breaker (c) occurs more often when the wind is blowing inward. It is convex on the back side and concave on the front. Of course it maintains its form for only a short time. It is this kind of wave that may form a "tube" within which a highly skilled surfer can travel for an extended distance (cover photo). Very large diameter tubes are called *mackers* ("You could drive a Mack truck through it.") In Fig. 2(a), white caps may be seen. These are sometimes referred to as breakers, but they are formed primarily by the wind blowing the tops off the waves. The body of the wave continues to advance undisturbed.

As the waves move through ever-shallower water on their path to the beach, their speed decreases and they may be seen to refract so that they become more and more parallel to the shore (see Fig. 3).

## Board Surfing

The area of the beach where the waves pitch up and collapse ("the surf zone") is determined by the swell direction and bottom contours. "The surf zone is established when the depth of the water is roughly equal to 1.3 times the wave height. This means a five foot wave will begin to break when the water is about six and a half feet deep."[13]

***Entering the run.*** Watching a surfer start a run is interesting, because the board and rider must somehow accelerate up to the wave velocity. First the surfer swims or paddles out beyond where the waves are breaking. From the shore you'll notice that all the surfers gather at roughly the same spot. Surfers report that with experience "you'll get to recognize the position where waves form and get an intuition that it's big enough to take you with it."[14] There are two modes of acceleration as a selected wave approaches — paddling forward as hard as possible, and then allowing the wave itself to accelerate you. A rough calculation in which drag forces are neglected illustrates the basic physics (see Fig. 4).

If the angle of the wave surface to the horizontal is $\theta$, the accelerating force is $F\sin\theta$, where $F$ is the force perpendicular to the water surface that the water exerts on the board. The vertical component is $F\cos\theta = mg$, where $m$ is the mass of the board and rider (it is actual-

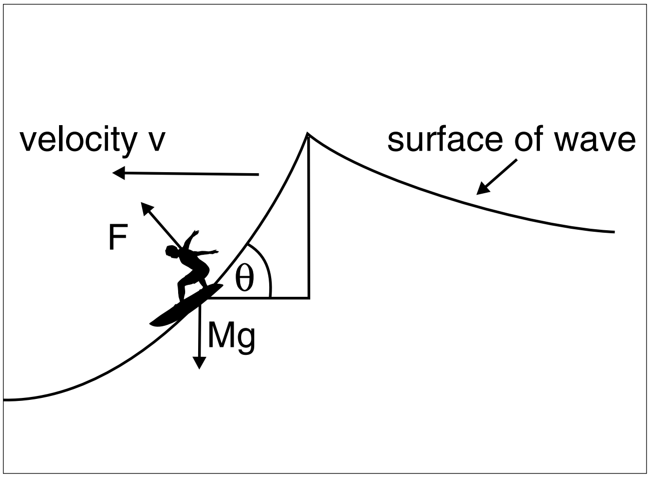


**Fig. 4. Acceleration of surfer at start of run.**

ly a little more than this because there is a positive vertical component to the acceleration). Let $t$ be the time over which the acceleration occurs. Then the momentum imparted is given by

$$m\Delta v = (F\sin\theta)t$$

or

$$m\Delta v = (mg\tan\theta)t$$

so that

$$\Delta v = (g\tan\theta)t.$$

If we assume that $t$ is a fraction $f$ of the period $T$ of the wave, we have $t = fT$, and since $T = \lambda/v$, we may write

$$\Delta v = v - v_o = f\lambda g\tan\theta.$$

As an example, let us insert the values of $f = 0.1$, $\lambda = 6.3$ m, $\theta = 60°$, and $v_o = 0$. Then $v = 3.5$ m/s, which is roughly equal to the required wave velocity.

After accelerating up to the wave speed, the rider rises to a standing position and begins rapidly skimming over the water surface, even though the average density of board and surfer combined is quite high. But the wave becomes steeper as it advances, and to avoid a spill the rider must move down the front of the wave to where the acceleration is less. This provides stability (similar to the "phase stability" necessary in accelerating a particle in a high-energy accelerator).

**Riding the Wave.** The main objective of the surfer is to ride the wave at the point (the "critical spot") that will carry him in with the greatest stability and highest speed possible. The actual distance it is possible to travel directly toward shore is relatively short, if the breaking cycle of the wave itself is short. This will depend on the height of the wave, the wavelength, and the shelving of the beach. If, instead of moving directly perpendicular to the wave front, the surfer moves in a path more parallel to the crest, the ride can be greatly extended. Figure 5 shows a situation in which the velocity components parallel and perpendicular to the crest are comparable. Clearly, too, a beach that shelves slowly will promote a long distance for the wave to break. The surfer must then start to ride the wave at the point where the water is starting to rise preparatory to breaking, just ahead of the vertical portion and under the breaking curl. "The ultimate is getting tubed, being in the space behind the falling curtain of water. Expert surfers get tubed a lot if the wave is right."[15] However, the breaking of the wave depends also on the angle it approaches the beach. Were the wave heading directly toward the beach, it would break at the same time all the way along its length. In fact, since the wave generally approaches the beach at a shallow angle (Fig. 3), the point at which it breaks runs along the crest, and if the surfer can move at this same speed, he can travel very rapidly parallel to the crest. The big question is, what keeps him going at these very high speeds? Clearly, he is gliding or sliding over, and not plowing through the water. The surfer is making use of the slope of the water surface and sliding down it, as a skier slides down a snowy slope. However, this is not the complete answer because he must also move sideways. As discussed below, a skilled surfer may exert considerable control over the direction of motion of the board.

**Mechanism of Surfboard.** Large oceangoing vessels displace a great deal of water, which moves around the hull as the ship plows through the sea. But a high-speed, suitably designed boat can "plane," rising up out of the water and gliding over the surface. Just so in the designs of surfboards. Early boards were quite long (10 ft or more) and usually made of plywood. Because of their large area, they experienced relatively large frictional forces as they moved over the water. Many of today's boards are shorter and very light, made of foam and covered with epoxy. The smaller the board, however, the faster it must move if the board and the surfer are to plane rather than sink.

**Fig. 5. Surfer, Josette Lagardere, with large component of velocity parallel to wave front.** (*photo by Martha Jenkins, seejanerunpictures.com*)

One or more "skegs" attached to the rear of the board act as rudders while the rider imparts impulses to the board by shifting his weight.

What kind of board should surfers use? At advanced levels, that's a question of interminable discussion. However, beginners starting with longboards (between 8 and 10 ft) can be "getting up and surfing it within an hour."[16] The more streamlined 6- or 7-ft "champion" boards are designed for speed and maneuverability and require much greater control skills. Surfboard design is an essential element in the study of the sport and could in itself be the topic of an entire article.

***Hanging Ten.*** An interesting physical problem associated with surfing is captured by the phrase "hang ten." Hanging ten involves having one's 10 toes over the front end of the surfboard. This is a trick that is not so common nowadays because for most people it requires a very heavy board, which is not readily available anymore. The old boards could weigh in excess of 100 lb.[17] Modeling the board/rider system as a simple seesaw, the weight of a rider at the front of the board could be balanced by the weight of the board acting at its center of mass. For a 100-lb rider hanging ten on a 100-lb, 8-ft board, a simple calculation of torques shows that rotational equilibrium occurs if the center of support provided by the wave is about two feet back from the front of the board. Of course, this explanation will not suffice for modern boards, which may weigh less than 8 lb. A very few highly skilled surfers are able to hang ten on a modern longboard. It requires allowing the wave to overtake the rider so that only the front of the board is out of the water at the wave front. Just as the wave is pitching forward, the rider runs to the tip and hangs ten for a brief period (see Fig. 6). Running forward causes the board to accelerate out of the wave and the rider must quickly move back.

## Comments

Surfboard riding, along with other water sports, has enjoyed an enormous spurt in popularity in the last 50 years. Nowadays, professional surf forecasters analyze weather maps and storm data to predict the quality of a wave's arrival

**Fig. 6. Mickey Muñoz "hanging ten."**

anywhere from three to 10 days in advance.[18] It's even possible to get an academic degree in Surfing Science.[19]

Students may enjoy learning about some of the basic physical concepts involved in surfing. But it is the ride — being caught up and hurled shoreward in what Jack London called a "wrestle with the sea" — that lures humans back to the beach time and again. "There is no other feeling like standing up on a surfboard and guiding it in a jumping, sliding rush across glassy water: speed, thrills, and fun. You'll know. You'll feel it. Surfers have an apt term for it: it's called *stoke*."[20]

## Acknowledgments

I would like to thank two anonymous referees whose knowledge of both physics and surfing contributed greatly to the improvement of this article.

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